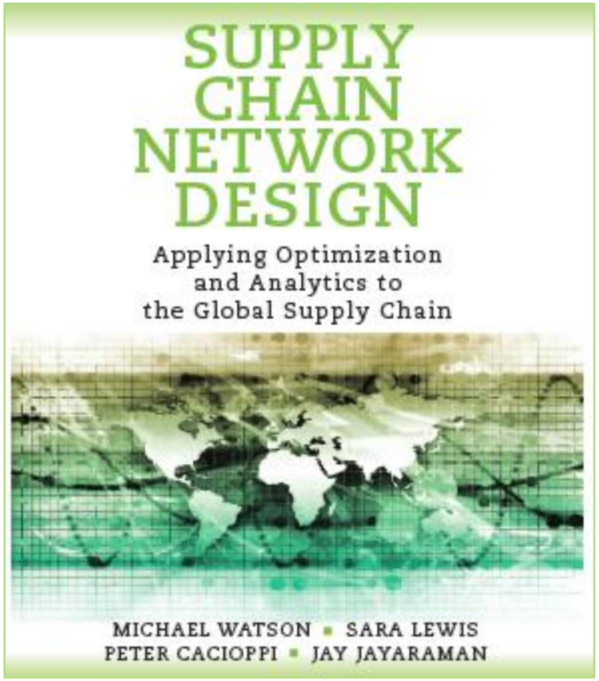
Chapter 4: Alternative Service Levels and Sensitivity Analysis



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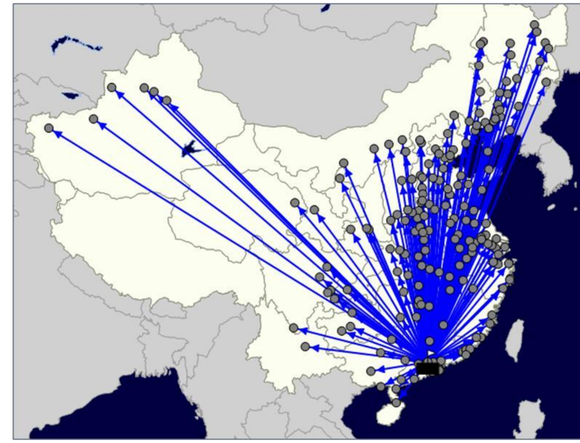
Network Design Service Level

- The term "Service Level" is used too much without definition
  - It has many different meanings
  - Unclear definitions do not lead to clear solutions
- Valid definitions for network design
  - Average distance to customers
  - Percent of customers within a certain distance
- Definitions not applicable to network design
  - Fill rate
  - Late orders
- Valid service level constraints
  - Maximum distance to a customer

Chen's Cosmetics – Corporate Background

- Chun Chen, a former makeup artist decided to team with her brother, a chemist to begin producing a line of affordable, professional quality cosmetics to be sold across China. Her new company was called Chen's Cosmetics.

- The company began with a single production facility in Guangzhou, China which served the entire customer base made up of distributor's warehouse locations across China



Existing Distribution from Chen's Plant to all Customers

- The cosmetic line immediately become very popular and Chen's now finds themselves with more demand than their existing plant can produce. As the CEO, Chun Chen decides they will add two new plants in China to alleviate this problem and offer better service to their customers.

- After some initial research, Chen knew that building a plant in any of their potential locations would cost just about the same and decided that their decision for locating these additional facilities should then be focused on locations which would enable to the best service to their existing customer base.

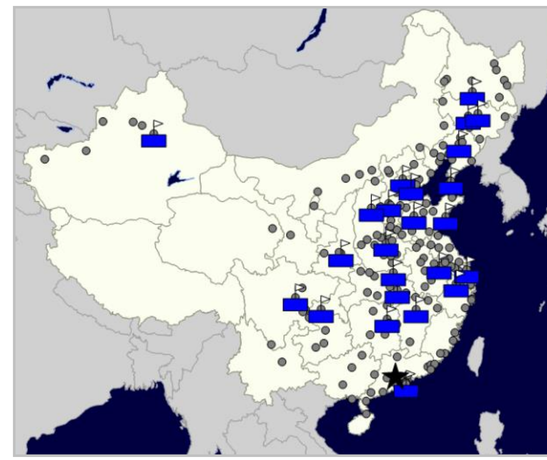
- The question was… how do they determine the location for these additional plants based soley on Service Level?

Chen’s Cosmetics – China Problem Definition

- The Logistics team at Chen’s Cosmetics decided to use a commercial Network Design application to find help them find the optimal answer to their question.

- The team generated a list of 24 potential locations where there was space and favorable conditions for building new plants. You can see these locations plotted as the light blue rectangles with flags on top within the network graphic below.

  - Knowing what they did about the constant cost of building a plant the team set out to focus the model on Service Level, which would require asking the model to **minimize the weighted average distance to all customers** or **maximize the percentage of customers within a certain distance** when selecting the optimal two plant locations



Dark Plant = Existing Guangzhou Location
Lights Plants = Potential Plant Locations

Chen’s Cosmetics Current Network Layered with 24 Potential sites where they could Locate their additional Plants

Math Formulation for Maximizing Demand within a Given Distance

- To maximize the demand within a given distance, we change our objective function to:

$$\textbf{Maximize} \quad \sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist\ ?\ 0 : 1) d_j Y_{i,j}$$

Didn't I say If-Statements are bad??

We are maximizing now

The objective still sums up over all customers and facilities

This is just an expression that says for every *i,j* combination, we test whether the distance is greater than *HighServiceDist*. If it is, we give it a 0. A 0 is bad b/c it says that if this combination is used it adds nothing to our objective (it adds zero). If it is within the *HighServiceDist*, then we get a 1 and if this combination is used it helps the objective function. This expression is nothing more than a matrix in Excel

In the end, we are just multiplying the demand by whether we made the assignment. If the previous term is a 0 the objective function gets no credit, so it has incentive to find a 1 from the HighServiceDist matrix

New Constraints for Service

- If you don’t specify any other constraints, the customers that are outside of the *HighServiceDist* will be assigned randomly (there is nothing to direct the optimization otherwise)
- The *AvgServiceDist* constraint can help:

$$\sum_{i\in I}\sum_{j\in J} dist_{i,j}\, d_j Y_{i,j} < AvgServiceDist \sum_{j\in J} d_j$$

This is the same as our objective when minimizing the total weighted distance. This is simply the distance multiplied by the demand for each assignment. (If we divide by the total demand, we get the weighted average distance.)

The *AvgServiceDist* is just a factor. This factor is then multiplied by the sum of the demand. The end result is a single number that can be compared to the total weighted distance (except here we are using a single number instead of the average).

If you set the *AvgServiceDist* factor relatively tight, this constraints will clean up the solution by making sure customers are assigned to the closest open warehouse.

Full Formulation

Maximize $$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist\ ?\ 0 : 1) d_j Y_{i,j}$$

Subject To:

$$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j} < AvgServiceDist \sum_{j \in J} d_j$$

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

$$\sum_{i \in I} X_i = P$$

$$Y_{i,j} \le X_i; \forall i \in I, \forall j \in J$$

$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

Setting a Maximum Distance Restriction

- Sometimes it is helpful to set an overall maximum distance for any customer (you can do this by types of customers if you want). Here is this family of constraints:

$$Y_{i,j} \le (dist_{i,j} > MaximumDist\,?\,0:1); \forall i \in I, \forall j \in J$$

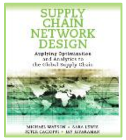
These constraints are going to control the legitimate values of $Y_{i,j}$

We read this just like the previous example. the *MaximumDist* is just a factor that we input. If the distance from i to j is greater than this factor (meaning it is further than we will allow), then we assign this term a 0. If it is less than or equal to it, then it is assigned a 1.

When it is assigned a 0, this constraint means that $Y_{i,j}$ must also be 0 (and thus not allowing customer j to be assigned to facility i). If it is a 1, it means that there are no restrictions on $Y_{i,j}$ because $Y_{i,j}$ can only be 0 or 1.

This indicates that we have a family of constraints. A constraint for every combination of i and j.

Formulation with Max Distance Constraint



Maximize $$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist\,?\,0:1) d_j Y_{i,j}$$

Subject to:

$$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j} < AvgServiceDist \sum_{j \in J} d_j$$

$$Y_{i,j} \le (dist_{i,j} > MaximumDist\,?\,0:1); \forall i \in I, \forall j \in J$$

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

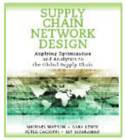
$$\sum_{i \in I} X_i = P$$

$$Y_{i,j} \le X_i; \forall i \in I, \forall j \in J$$

$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

Let’s See What this Looks Like in PuLP

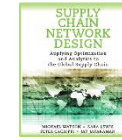



```
# The objective function is added to 'prob' first
demand_within_distance = lpSum([high_service_dist_par[w,c]*customer_demands[c]*assign_vars[w,c]
                    for w in warehouses for c in customers])
```

This line of code, which is in the data block, shows the “if-then” statement

```
# Setting the value to be 1 if customer c is within the given high service distance of warehouse w
high_service_dist_par = {(w,c): 1 if distance[w,c] <= high_service_dist else 0 for w in warehouses for c in customers}
```

Let's Run the 1st Model in PuLP



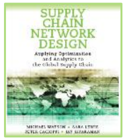
In the last block of code, let's keep the 2nd model commented out….

```
df_assign_vars_1, list_facility_vars_1 = optimal_location_1(high_service_dist_par,
# df_assign_vars_2, list_facility_vars_2 = optimal_location_2(high_service_dist_pa
```

Let's set the defaults to the following… And, then Run-All

```
# Change the service distance (in km) you'd
high_service_dist = 600
# Change the average service distance of all
avg_service_dist = 1000
# Maximum distance (in km) a customer can be
maximum_dist = 5000
# Quantity of demand to be serviced within t
high_service_demand = 500100100  #start with
# Change the number of warehouses you would
number_of_whs = 3  #<<<------ you can change
```

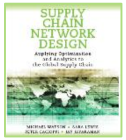
Model #1 Results




```
Model_1
Optimization Status Optimal
Objective(Demand within Max Service): 131,645,389.0
% of Demand within  600 km:  66.06 %
Average service distance: 658.5  km
Run Time of model 1 in seconds 0.9
```

Remember, how are customers assigned to warehouses? (those within the max service and those outside)?

Clean Up Model: Full Formulation Showing the HighServiceDist as a Constraint



Minimize $$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j}$$

Subject To:

$$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist ? 0 : 1) d_j Y_{i,j} \geq HighServiceDemand$$

$$Y_{i,j} \leq (dist_{i,j} > MaximumDist ? 0 : 1); \forall i \in I, \forall j \in J$$

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

$$\sum_{i \in I} X_i = P$$

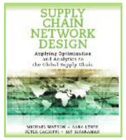
$$Y_{i,j} \leq X_i; \forall i \in I, \forall j \in J$$

$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

How does this model work to “clean up” the previous model?

Let’s Activate the Clean Up Model



In the last block of code, activate the 2nd model

```
df_assign_vars_1, list_facility_vars_1 = optimal_location_1(high_service_dist_pa

df_assign_vars_2, list_facility_vars_2 = optimal_location_2(high_service_dist_pa
```

There is a key parameter we need to change here...

```
# Change the service distance (in km) you'd

high_service_dist = 600

# Change the average service distance of all

avg_service_dist = 1000

# Maximum distance (in km) a customer can be

maximum_dist = 5000

# Quantity of demand to be serviced within t

high_service_demand = 500100100  #start with

# Change the number of warehouses you would

number_of_whs = 3  #<<<------ you can change
```

Let's Review the 2nd Model

The objective function

```
# The objective function is added to 'prob' first
total_distance = lpSum([distance[w,c]*customer_demands[c]*assign_vars[w,c]
                        for w in warehouses for c in customers])
```

This is the key constraint to relate back to 1st Model

```
# Ensures that a certain demand is within the service distance
chens_problem_2 += LpConstraint(e = lpSum([high_service_dist_par[w,c]*customer_demands[c]*assign_vars[w,c] for
                                sense=LpConstraintGE,
                                name="_Avg_Served",
                                rhs=high_service_demand)
```

What is the parameter you need to update??

Now "Run All"

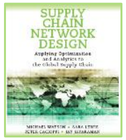
Let's Review the 2nd Model

```
Model_1
Optimization Status Optimal
Objective(Demand within Max Service): 131,645,389.0
% of Demand within  600 km:  66.06 %
Average service distance: 658.5  km
Run Time of model 1 in seconds 0.9

Model_2
Optimization Status Optimal
Objective(Total Demand-Distance):  123834216789.27011
% of Demand within  600 km:  66.06 %
Equivalent Objective(Average service distance):  621.4 km
Run Time of model 2 in seconds 1.2
```

In Model 2, How did it the % of Demand within 600 km stay the same??

Model Clean Up Steps-- Review



1. Run this model and note the answer (as *HighServiceDemand*). This is the most demand you can meet in the service distance.

Maximize $$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist\,?\,0:1) d_j Y_{i,j}$$

Subject to:

$$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j} < AvgServiceDist \sum_{j \in J} d_j$$

$$Y_{i,j} \le (dist_{i,j} > MaximumDist\,?\,0:1); \forall i \in I, \forall j \in J$$

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$$X_i \in \{0,1\}; \forall i \in I$$

2. Now run this model. The constraint basically makes sure that the same facilities are picked and they have the same assignments as before (barring multiple optimal solutions). But, the objective will assign all the remaining demand to the closest facility.

Minimize $$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j}$$

Subject To:

$$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist\,?\,0:1) d_j Y_{i,j} \ge HighServiceDemand$$

$$Y_{i,j} \le (dist_{i,j} > MaximumDist\,?\,0:1); \forall i \in I, \forall j \in J$$

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

$$\sum_{i \in I} X_i = P$$

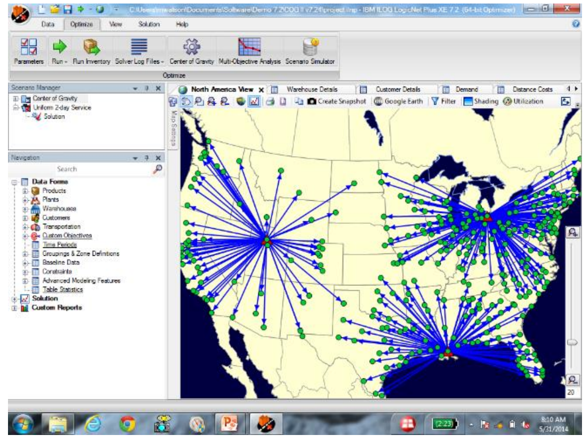
$$Y_{i,j} \le X_i; \forall i \in I, \forall j \in J$$

$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

How Would You Compare These Models?

Remember This Problem?



Ealier, we used this simple model. We tricked the distance, up to 800 miles, we used \$1/mile after 800 we used \$10/mile. We manipulated the distance matrix.

Minimize $$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j}$$

Subject to:

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

$$\sum_{i \in I} X_i = P$$

$$Y_{i,j} \le X_i; \forall i \in I, \forall j \in J$$

$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

versus

Maximize $$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist ? 0 : 1) d_j Y_{i,j}$$

Subject to:

$$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j} < AvgServiceDist \sum_{j \in J} d_j$$

$$Y_{i,j} \le (dist_{i,j} > MaximumDist ? 0 : 1); \forall i \in I, \forall j \in J$$

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

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$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

The model we just covered

Minimize $$\sum_{i \in I} \sum_{j \in J} dist_{i,j} d_j Y_{i,j}$$

Subject To:

$$\sum_{i \in I} \sum_{j \in J} (dist_{i,j} > HighServiceDist ? 0 : 1) d_j Y_{i,j} \ge HighServiceDemand$$

$$Y_{i,j} \le (dist_{i,j} > MaximumDist ? 0 : 1); \forall i \in I, \forall j \in J$$

$$\sum_{i \in I} Y_{i,j} = 1; \forall j \in J$$

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$$Y_{i,j} \le X_i; \forall i \in I, \forall j \in J$$

$$Y_{i,j} \in \{0,1\}; \forall i \in I, \forall j \in J$$

$$X_i \in \{0,1\}; \forall i \in I$$

How would you compare and contrast these approaches?

Work-around, but easy model; No work-around, but a little harder